The inscription reads, "A. Leonardo Fibonacci, Insigne Matematico Piisano del Secolo XII."
Photo by Robert R. Prechter, Sr.

Historical And Mathematical Background Of The Wave Principle

The Fibonacci (pronounced fib-eh-nahr'-chee) sequence of numbers was discovered (actually rediscovered) by Leonardo Fibonacci da Pisa, a thirteenth century mathematician. We will outline the historical background of this amazing man and then discuss more fully the sequence (technically it is a sequence and not a series) of numbers that bears his name. When Elliott wrote Nature's Law, he referred specifically to the Fibonacci sequence as the mathematical basis for the Wave Principle. It is sufficient to state at this point that the stock market has a propensity to demonstrate a form that can be aligned with the form present in the Fibonacci sequence. (For a further discussion of the mathematics behind the Wave Principle, see "Mathematical Basis of Wave Theory," by Walter E. White, in New Classics Library's forthcoming book.)

In the early 1200s, Leonardo Fibonacci of Pisa, Italy published his famous Liber Abacci (Book of Calculation) which introduced to Europe one of the greatest mathematical discoveries of all time, namely the decimal system, including the positioning of zero as the first digit in the notation of the number scale. This system, which included the familiar symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9, became known as the Hindu-Arabic system, which is now universally used.

Under a true digital or place-value system, the actual value represented by any symbol placed in a row along with other symbols depends not only on its basic numerical value but also on its position in the row, i.e., 58 has a different value from 85. Though thousands of years earlier the Babylonians and Mayas of Central America separately had developed digital or place-value systems of numeration, their methods were awkward in other respects. For this reason, the Babylonian system, which had been the first to use zero and place values, was never carried forward into the mathematical systems of Greece, or even Rome, whose numeration comprised the seven symbols I, V, X, L, C, D, and M, with non-digital values assigned to those symbols. Addition, subtraction, multiplication and division in a system using these non-digital symbols is not an easy task, especially when large numbers are involved. Paradoxically, to overcome this problem, the Romans used the very ancient digital device known as the abacus. Because this instrument is digitally based and contains the zero principle, it functioned as a necessary supplement to the Roman computational system. Throughout the ages, bookkeepers and merchants depended on it to assist them in the mechanics of their tasks. Fibonacci, after expressing the basic principle of the abacus in Liber Abacci, started to use his new system during his travels. Through his efforts, the new system, with its easy method of calculation, was eventually transmitted to Europe. Gradually the old usage of Roman numerals was replaced with the Arabic numeral system. The introduction of the new system to Europe was the first important achievement in the field of mathematics since the fall of Rome over seven hundred years before. Fibonacci not only kept mathematics alive during the Middle Ages, but laid the foundation for great developments in the field of higher mathematics and the related fields of physics, astronomy and engineering.

Although the world later almost lost sight of Fibonacci, he was unquestionably a man of his time. His fame was such that Frederick II, a scientist and scholar in his own right, sought him out by arranging a visit to Pisa. Frederick II was Emperor of the Holy Roman Empire, the King of Sicily and Jerusalem, scion of two of the noblest families in Europe and Sicily, and the most powerful prince of his day. His ideas were those of an absolute monarch, and he surrounded himself with all the pomp of a Roman emperor.

The meeting between Fibonacci and Frederick II took place in 1225 A.D. and was an event of great importance to the town of Pisa. The Emperor rode at the head of a long procession of trumpeters, courtiers, knights, officials and a menagerie of animals. Some of the problems the Emperor placed before the famous mathematician are detailed in Liber Abacci. Fibonacci apparently solved the problems posed by the Emperor and forever more was welcome at the King's Court. When Fibonacci revised Liber Abacci in 1228 A.D., he dedicated the revised edition to Frederick II.

It is almost an understatement to say that Leonardo Fibonacci was the greatest mathematician of the Middle Ages. In all, he wrote three major mathematical works: the Liber Abacci, published in 1202 and revised in 1228, Practica Geometriae, published in 1220, and Liber Quadratorum. The admiring citizens of Pisa documented in 1240 A.D. that he was "a discreet and learned man," and very recently Joseph Gies, a senior editor of the Encyclopedia Britannica, stated that future scholars will in time "give Leonard of Pisa his due as one of the world's great intellectual pioneers." His works, after all these years, are only now being translated from Latin into English. For those interested, the book entitled Leonard of Pisa and the New Mathematics of the Middle Ages, by Joseph and Frances Gies, is an excellent treatise on the age of Fibonacci and his works.

Although he was the greatest mathematician of medieval times, Fibonacci's only monuments are a statue across the Arno River from the Leaning Tower and two streets which bear his name, one in Pisa and the other in Florence. It seems strange that so few visitors to the 179-foot marble Tower of Pisa have ever heard of Fibonacci or seen his statue. Fibonacci was a contemporary of Bonanna, the architect of the Tower, who started building in 1174 A.D. Both men made contributions to the world, but the one whose influence far exceeds the other's is almost unknown.

The Fibonacci Sequence

In Liber Abacci, a problem is posed that gives rise to the sequence of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, and so on to infinity, known today as the Fibonacci sequence. The problem is this:

How many pairs of rabbits placed in an enclosed area can be produced in a single year from one pair of rabbits if each pair gives birth to a new pair each month starting with the second month?

In arriving at the solution, we find that each pair, including the first pair, needs a month's time to mature, but once in production, begets a new pair each month. The number of pairs is the same at the beginning of each of the first two months, so the sequence is 1, 1. This first pair finally doubles its number during the second month, so that there are two pairs at the beginning of the third month. Of these, the older pair begets a third pair the following month so that at the beginning of the fourth month, the sequence expands 1, 1, 2, 3. Of these three, the two older pairs reproduce, but not the youngest pair, so the number of rabbit pairs expands to five. The next month, three pairs reproduce so the sequence expands to 1, 1, 2, 3, 5, 8 and so forth. Figure 3-1 shows the Rabbit Family Tree with the family growing with logarithmic acceleration. Continue the sequence for a few years and the numbers become astronomical. In 100 months, for instance, we would have to contend with 354,224,848,179,261,915,075 pairs of rabbits.

The Fibonacci sequence resulting from the rabbit problem has many interesting properties and reflects an almost constant relationship among its components.

The Rabbit Family Tree

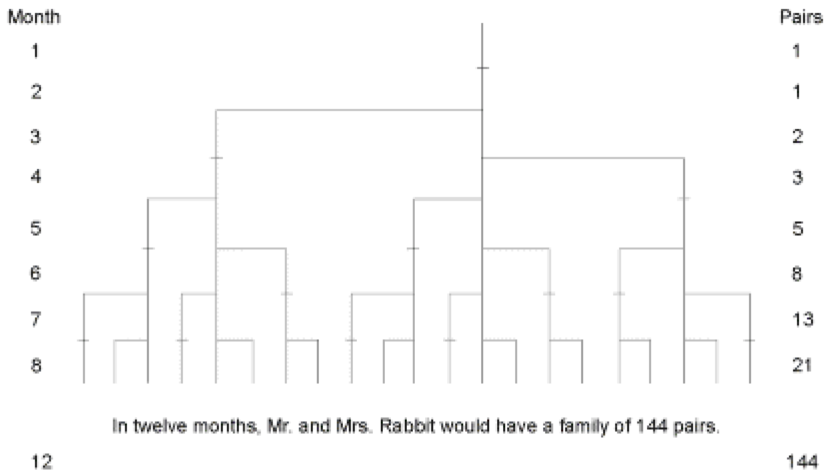


Figure 3-1

The sum of any two adjacent numbers in the sequence forms the next higher number in the sequence, viz., 1 plus 1 equals 2, 1 plus 2 equals 3, 2 plus 3 equals 5, 3 plus 5 equals 8, and so on to infinity.

The Golden Ratio

After the first several numbers in the sequence, the ratio of any number to the next higher is approximately .618 to 1 and to the next lower number approximately 1.618 to 1. The further along the sequence, the closer the ratio approaches phi (denoted f) which is an irrational number, .618034....

Between alternate numbers in the sequence, the ratio is approximately .382, whose inverse is 2.618. Refer to Figure 3-2 for a ratio table interlocking all Fibonacci numbers from 1 to 144.

Fibonacci Ratio Table

DENOMINATOR \ NUMERATOR	1	2	3	5	8	13	21	34	55	89	144
1	1.00	2.00	3.00	5.00	8.00	13.00	21.00	34.00	55.00	89.00	144.00
2	.50	1.00	1.50	2.50	4.00	6.50	10.50	17.00	27.50	44.50	72.00
3	.333	.667	1.00	1.667	2.667	4.33	7.00	11.33	18.33	29.67	48.00
5	.20	.40	.60	1.00	1.60	2.60	4.20	6.80	11.00	17.80	28.80
8	.125	.25	.375	.625	1.00	1.625	2.625	4.25	6.875	11.125	18.00
13	.077	.154	.231	.385	.615	1.00	1.615	2.615	4.23	6.846	11.077
21	.0476	.0952	.1429	.238	.381	.619	1.00	1.619	2.619	4.238	6.857
34	.0294	.0588	.0882	.147	.235	.3824	.6176	1.00	1.618	2.618	4.235
55	.01818	.03636	.0545	.0909	.1455	.236	.3818	.618	1.00	1.618	2.618
89	.011236	.02247	.0337	.05618	.08989	.146	.236	.382	.618	1.00	1.618
144	.006944	.013889	.0208	.0347	.05556	.0903	.1458	.236	.382	.618	1.00

Toward perfect ratios

Figure 3-2

Phi is the only number that when added to 1 yields its inverse: .618 + 1 = 1 ч .618. This alliance of the additive and the multiplicative produces the following sequence of equations: .6182 = 1 - .618, .6183 = .618 - .6182, .6184 = .6182 - .6183, .6185 = .6183 - .6184, etc. or alternatively, 1.6182 = 1 + 1.618, 1.6183 = 1.618 + 1.6182, 1.6184 = 1.6182 + 1.6183, 1.6185 = 1.6183 + 1.6184, etc.

Some statements of the interrelated properties of these four main ratios can be listed as follows:

1.618 - .618 = 1,
1.618 x .618 = 1,
1 - .618 = .382,
.618 x .618 = .382,
2.618 - 1.618 = 1,
2.618 x .382 = 1,
2.618 x .618 = 1.618,
1.618 x 1.618 = 2.618.

Besides 1 and 2, any Fibonacci number multiplied by four, when added to a selected Fibonacci number, gives another Fibo-nacci number, so that:

3 x 4 = 12; + 1 = 13,
5 x 4 = 20; + 1 = 21,
8 x 4 = 32; + 2 = 34,
13 x 4 = 52; + 3 = 55,
21 x 4 = 84; + 5 = 89,
and so on.

As the new sequence progresses, a third sequence begins in those numbers that are added to the 4x multiple. This relationship is possible because the ratio between second alternate Fibonacci numbers is 4.236, where .236 is both its inverse and its difference from the number 4. This continuous series-building property is reflected at other multiples for the same reasons. 1.618 (or .618) is known as the Golden Ratio or Golden Mean. Its proportions are pleasing to the eye and an important phenomenon in music, art, architecture and biology. William Hoffer, writing for the December 1975 Smithsonian Magazine, said: ...the proportion of .618034 to 1 is the mathematical basis for the shape of playing cards and the Parthenon, sunflowers and snail shells, Greek vases and the spiral galaxies of outer space. The Greeks based much of their art and architecture upon this proportion.

They called it "the golden mean."

Fibonacci's abracadabric rabbits pop up in the most unexpected places. The numbers are unquestionably part of a mystical natural harmony that feels good, looks good and even sounds good. Music, for example, is based on the 8 note octave. On the piano this is represented by 8 white keys, 5 black ones — 13 in all. It is no accident that the musical harmony that seems to give the ear its greatest satisfaction is the major sixth. The note E vibrates at a ratio of .62500 to the note C. A mere .006966 away from the exact golden mean, the proportions of the major sixth set off good vibrations in the cochlea of the inner ear — an organ that just happens to be shaped in a logarithmic spiral.

The continual occurrence of Fibonacci numbers and the golden spiral in nature explains precisely why the proportion of .618034 to 1 is so pleasing in art. Man can see the image of life in art that is based on the golden mean.

Nature uses the Golden Ratio in its most intimate building blocks and in its most advanced patterns, in forms as minuscule as atomic structure, microtubules in the brain and DNA molecules to those as large as planetary orbits and galaxies. It is involved in such diverse phenomena as quasi crystal arrangements, planetary distances and periods, reflections of light beams on glass, the brain and nervous system, musical arrangement, and the structures of plants and animals. Science is rapidly demonstrating that there is indeed a basic proportional principle of nature. By the way, you are holding your mouse with your five appendages, all but one of which have three jointed parts, five digits at the end, and three jointed sections to each digit.

Lesson 17: Fibonacci Geometry

The Golden Section

Any length can be divided in such a way that the ratio between the smaller part and the larger part is equivalent to the ratio between the larger part and the whole (see Figure 3-3). That ratio is always .618.

Figure 3-3

The Golden Section occurs throughout nature. In fact, the human body is a tapestry of Golden Sections (see Figure 3-9) in everything from outer dimensions to facial arrangement. "Plato, in his Timaeus," says Peter Tompkins, "went so far as to consider phi, and the resulting Golden Section proportion, the most binding of all mathematical relations, and considered it the key to the physics of the cosmos." In the sixteenth century, Johannes Kepler, in writing about the Golden, or "Divine Section," said that it described virtually all of creation and specifically symbolized God's creation of "like from like." Man is the divided at the navel into Fibonacci proportions. The statistical average is approximately .618. The ratio holds true separately for men, and separately for women, a fine symbol of the creation of "like from like." Is all of mankind's progress also a creation of "like from like?"

The Golden Rectangle

The sides of a Golden Rectangle are in the proportion of 1.618 to 1. To construct a Golden Rectangle, start with a square of 2 units by 2 units and draw a line from the midpoint of one side of the square to one of the corners formed by the opposite side as shown in Figure 3-4.

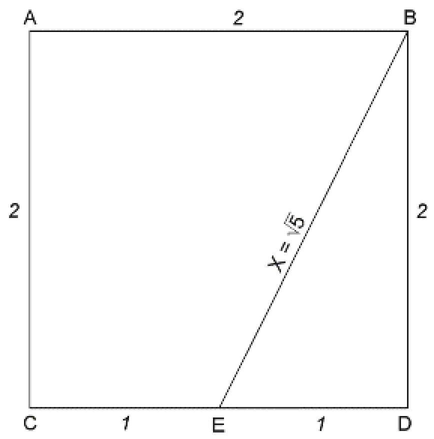


Figure 3-4

Triangle EDB is a right-angled triangle. Pythagoras, around 550 B.C., proved that the square of the hypotenuse (X) of a right-angled triangle equals the sum of the squares of the other two sides. In this case, therefore, X2 = 22 + 12, or X2 = 5. The length of the line EB, then, must be the square root of 5. The next step in the construction of a Golden Rectangle is to extend the line CD, making EG equal to the square root of 5, or 2.236, units in length, as shown in Figure

3-5. When completed, the sides of the rectangles are in the proportion of the Golden Ratio, so both the rectangle AFGC and BFGD are Golden Rectangles.

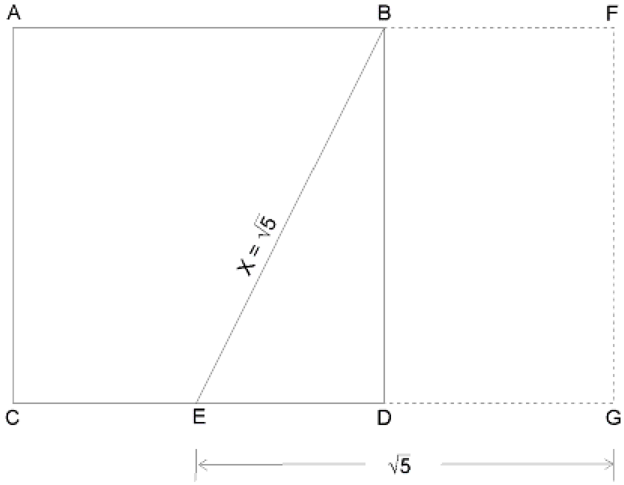


Figure 3-5

Since the sides of the rectangles are in the proportion of the Golden Ratio, then the rectangles are, by definition, Golden Rectangles.

Works of art have been greatly enhanced with knowledge of the Golden Rectangle. Fascination with its value and use was particularly strong in ancient Egypt and Greece and during the Renaissance, all high points of civilization. Leonardo da Vinci attributed great meaning to the Golden Ratio. He also found it pleasing in its proportions and said, "If a thing does not have the right look, it does not work." Many of his paintings had the right look because he used the Golden Section to enhance their appeal.

While it has been used consciously and deliberately by artists and architects for their own reasons, the phi proportion apparently does have an effect upon the viewer of forms. Experimenters have determined that people find the .618 proportion aesthetically pleasing. For instance, subjects have been asked to choose one rectangle from a group of different types of rectangles with the average choice generally found to be close to the Golden Rectangle shape. When asked to cross one bar with another in a way they liked best, subjects generally used one to divide the other into the phi proportion. Windows, picture frames, buildings, books and cemetery crosses often approximate Golden Rectangles.

As with the Golden Section, the value of the Golden Rectangle is hardly limited to beauty, but serves function as well. Among numerous examples, the most striking is that the double helix of DNA itself creates precise Golden Sections at regular intervals of its twists (see Figure 3-9).

While the Golden Section and the Golden Rectangle represent static forms of natural and man-made aesthetic beauty and function, the representation of an aesthetically pleasing dynamism, an orderly progression of growth or progress, can be made only by one of the most remarkable forms in the universe, the Golden Spiral.

The Golden Spiral

A Golden Rectangle can be used to construct a Golden Spiral. Any Golden Rectangle, as in Figure 3-5, can be divided into a square and a smaller Golden Rectangle, as shown in Figure 3-6. This process then theoretically can be continued to infinity. The resulting squares we have drawn, which appear to be whirling inward, are marked A, B, C, D, E, F and G.

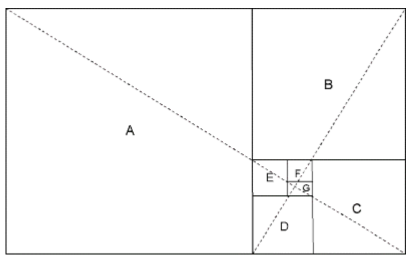


Figure 3-6

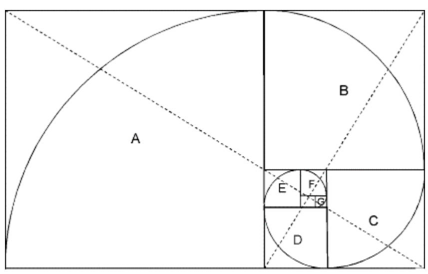
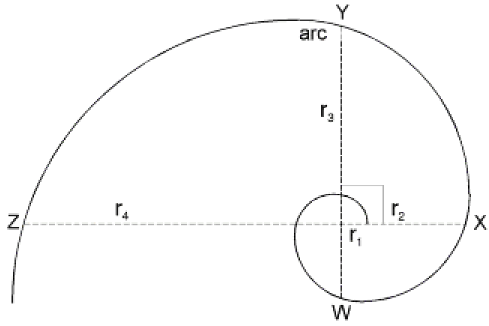


Figure 3-7

The dotted lines, which are themselves in golden proportion to each other, diagonally bisect the rectangles and pinpoint the theoretical center of the whirling squares. From near this central point, we can draw the spiral as shown in Figure 3-7 by connecting the points of intersection for each whirling square, in order of increasing size. As the squares whirl inward and outward, their connecting points trace out a Golden Spiral. The same process, but using a sequence of whirling triangles, also can be used to construct a Golden Spiral.

At any point in the evolution of the Golden Spiral, the ratio of the length of the arc to its diameter is 1.618. The diameter and radius, in turn, are related by 1.618 to the diameter and radius 90° away, as illustrated in Figure 3-8.



$$\frac{r_2}{r_1} = \frac{r_3}{r_2} = \frac{r_4}{r_3} = \ldots = \frac{r_n}{r_{n-1}} = 1.618$$

$$\frac{d_2}{d_1} = \frac{d_3}{d_2} = \ldots = \frac{d_n}{d_{n-1}} = 1.618$$

(where $d_1 = r_1 + r_3$, $d_2 = r_2 + r_4$, etc.)

$$\frac{\text{arcXY}}{\text{arcWX}} = \frac{\text{arcYZ}}{\text{arcXY}}\text{, etc.} = \frac{\text{arcXZ}}{\text{arcWY}} = 1.618$$

$$\frac{\text{arcWY}}{\text{diam. (WY)}} = \frac{\text{arcXZ}}{\text{diam. (XZ)}}\text{, etc.} = 1.618$$

Figure 3-8

The Golden Spiral, which is a type of logarithmic or equiangular spiral, has no boundaries and is a constant shape. From any point on the spiral, one can travel infinitely in either the outward or inward direction. The center is never met, and the outward reach is unlimited. The core of a logarithmic spiral seen through a microscope would have the same look as its widest viewable reach from light years away. As David Bergamini, writing for Mathematics (in Time-Life Books' Science Library series) points out, the tail of a comet curves away from the sun in a logarithmic spiral. The epeira spider spins its web into a logarithmic spiral. Bacteria grow at an accelerating rate that can be plotted along a logarithmic spiral. Meteorites, when they rupture the surface of the Earth, cause depressions that correspond to a logarithmic spiral. Pine cones, sea horses, snail shells, mollusk shells, ocean waves, ferns, animal horns and the arrange- ment of seed curves on sunflowers and daisies all form logarithmic spirals. Hurricane clouds and the galaxies of outer space swirl in logarithmic spirals. Even the human finger, which is composed of three bones in Golden Section to one another, takes the spiral shape of the dying poinsettia leaf when curled. In Figure 3-9, we see a reflection of this cosmic influence in numerous forms. Eons of time and light years of space separate the pine cone and the spiraling galaxy, but the design is the same: a 1.618 ratio, perhaps the primary law governing dynamic natural phenomena. Thus, the Golden Spiral spreads before us in symbolic form as one of nature's grand designs, the image of life in endless expansion and contraction, a static law governing a dynamic process, the within and the without sustained by the 1.618 ratio, the Golden Mean.

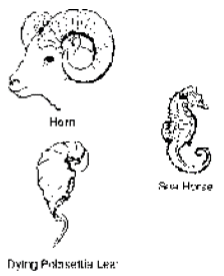


Figure 3-9a

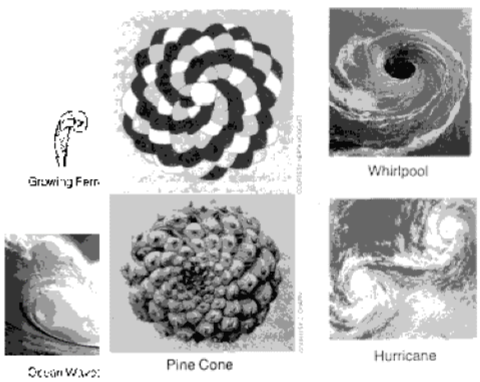


Figure 3-9b

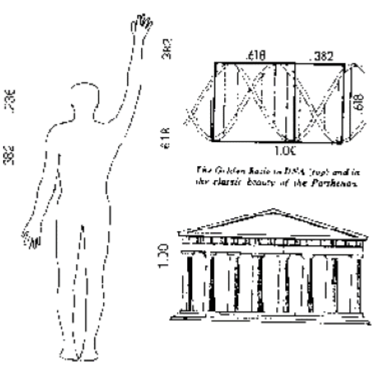


Figure 3-9c

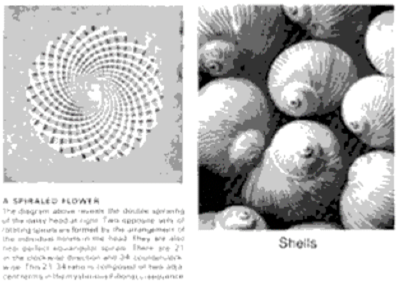


Figure 3-9d

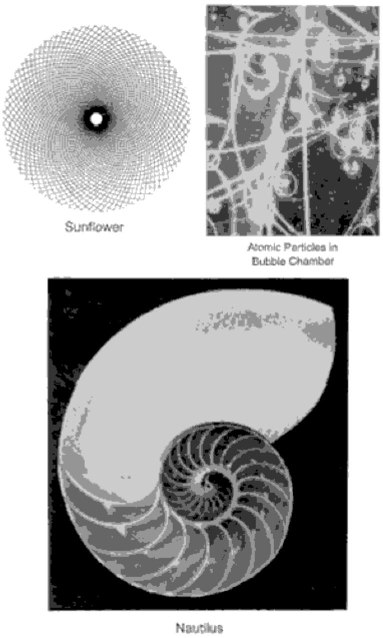


Figure 3-9e

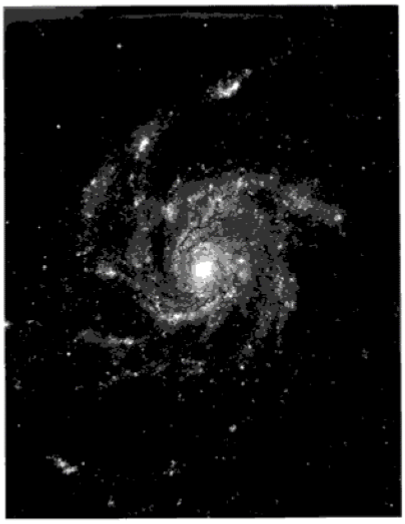


Figure 3-9f

Lesson 18: The Meaning Of Phi

The value of this ubiquitous phenomenon was deeply understood and profoundly appreciated by the greatest intellects of the ages. History abounds with examples of exceptionally learned men who held a special fascination for this mathematical formulation. Pythagoras chose the five-pointed star, in which every segment is in golden ratio to the next smaller segment, as the symbol of his Order; celebrated 17th century mathematician Jacob Bernoulli had the Golden Spiral etched into his headstone; Isaac Newton had the same spiral carved on the headboard of his bed (owned today by the Gravity Foundation, New Boston, NH). The earliest known aficionados were the architects of the Gizeh pyramid in Egypt, who recorded the knowledge of phi in its construction nearly 5000 years ago. Egyptian engineers consciously incorporated the Golden Ratio in the Great Pyramid by giving its faces a slope height equal to 1.618 times half its base, so that the vertical height of the pyramid is at the same time the square root of 1.618 times half its base. According to Peter Tompkins, author of Secrets of the Great Pyramid (Harper & Row, 1971), "This relation shows Herodotus' report to be indeed correct, in that the square of the height of the pyramid is Цf x Цf = f, and the areas of the face 1 x f = f."

Furthermore, using these proportions, the Egyptian scientists (apparently in order to build a scale model of the Northern Hemisphere) used pi and phi in an approach so mathematically sophisticated that it accomplished the feat of squaring the circle and cubing the sphere (i.e., making them of equal area and volume), a feat which was not duplicated for well over four thousand years.

While the mere mention of the Great Pyramid may serve as an engraved invitation to skepticism (perhaps for good reason), keep in mind that its form reflects the same fascination held by pillars of Western scientific, mathematical, artistic and philosophic thought, including Plato, Pythagoras, Bernoulli, Kepler, DaVinci and Newton. Those who designed and built the pyramid were likewise demonstrably brilliant scientists, astronomers, mathematicians and engineers. Clearly they wanted to enshrine for millennia the Golden Ratio as something of transcendent importance. That such a caliber of people, who were later joined by some of the greatest minds of Greece and the Enlightenment in their fascination for this ratio, undertook this task is itself important. As for why, all we have is conjecture from a few authors. Yet that conjecture, however obtuse, curiously pertains to our own observations. It has been surmised that the Great Pyramid, for centuries after it was built, was used as a temple of initiation for those who proved themselves worthy of understanding the great universal secrets. Only those who could rise above the crude acceptance of things as they seemed to discover what, in actuality, they were, could be instructed in "the mysteries," i.e., the complex truths of eternal order and growth.

Did such "mysteries" include phi? Tompkins explains, "The pharaonic Egyptians, says Schwaller de Lubicz, considered phi not as a number, but as a symbol of the creative function, or of reproduction in an endless series. To them it represented `the fire of life, the male action of sperm, the logos [referenced in] the gospel of St. John.'" Logos, a Greek word, was defined variously by Heraclitus and subsequent pagan, Jewish and Christian philosophers as meaning the rational order of the universe, an immanent natural law, a life-giving force hidden within things, the universal structural force governing and permeating the world.

Consider when reading such deep yet vague descriptions that these people could not clearly see what they sensed. They did not have graphs and the Wave Principle to make nature's growth pattern manifest and were doing the best they could to describe an organizational principle that they discerned as shaping the natural world. If these ancient philosophers were right that a universal structural force governs and permeates the world, should it not govern and permeate the world of man? If forms throughout the universe, including man's body, brain and DNA, reflect the form of phi, might man's activities reflect it as well? If phi is the life-force in the universe, might it be the impulse behind the progress in man's productive capacity? If phi is a symbol of the creative function, might it govern the creative activity of man? If man's progress is based upon production and reproduction "in an endless series," is it not reasonable that such progress has the spiraling form of phi, and that this form is discernible in the movement of the valuation of his productive capacity, i.e., the stock market? Just as the initiated Egyptians learned the hidden truths of order and growth in the universe behind the apparent randomness and chaos (something that modern "chaos theory" has finally rediscovered in the 1980s), so the stock market, in our opinion, can be understood properly if it is taken for what it is rather than for what it crudely appears to be upon cursory consideration. The stock market is not a random, formless mess reacting to current news events but a remarkably precise recording of the formal structure of the progress of man.

Compare this concept with astronomer William Kingsland's words in The Great Pyramid in Fact and in Theory that Egyptian astronomy/astrology was a "profoundly esoteric science connected with the great cycles of man's evolution." The Wave Principle explains the great cycles of man's evolution and reveals how and why they unfold as they do. Moreover, it encompasses micro as well as macro scales, all of which are based upon a paradoxical principle of dynamism and variation within an unaltered form.

It is this form that gives structure and unity to the universe. Nothing in nature suggests that life is disorderly or formless. The word "universe" means "one order." If life has form, then we must not reject the probability that human progress, which is part of the reality of life, also has order and form.

By extension, the stock market, which values man's productive enterprise, will have order and form as well. All technical approaches to understanding the stock market depend on the basic principle of order and form. Elliott's theory, however, goes beyond all others. It postulates that no matter how minute or how large the form, the basic design remains constant.

Elliott, in his second monograph, used the title Nature's Law — The Secret of the Universe in preference to "The Wave Principle" and applied it to all sorts of human activity. Elliott may have gone too far in saying that the Wave Principle was the secret of the universe, as nature appears to have created numerous forms and processes, not just one simple design. Nevertheless, some of history's greatest scientists, mentioned earlier, would probably have agreed with Elliott's formulation. At minimum, it is credible to say that the Wave Principle is one of the most important secrets of the universe. Even this grandiose claim at first may appear to be only so much tall talk to practically-minded investors, and quite understandably so.

The grand nature of the concept stretches the imagination and confounds the intellect, while its applicability is as yet unclear. First we must ask, can we both theorize and observe that there is indeed a principle that operates on the same mathematical basis in the heavens and earth as it does in the stock market?

The answer is yes. The stock market has the very same mathematical base as do these natural phenomena. The idealized Elliott concept of the progression of the stock market is an excellent base from which to construct the Golden Spiral, as Figure 3-10 illustrates with a rough approximation. In this construction, the top of each successive wave of higher degree is the touch point of the logarithmic expansion.

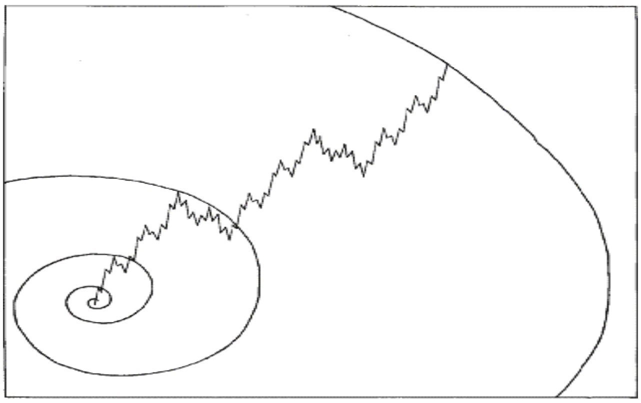


Figure 3-10

This result is possible because at every degree of stock market activity, a bull market subdivides into five waves and a bear market subdivides into three waves, giving us the 5-3 relationship that is the mathematical basis of the Elliott Wave Principle. We can generate the complete Fibonacci sequence, as we first did in Figure 1-4, by using Elliott's concept of the progression of the market. If we start with the simplest expression of the concept of a bear swing, we get one straight line decline. A bull swing, in its simplest form, is one straight line advance. A complete cycle is two lines. In the next degree of complexity, the corresponding numbers are 3, 5 and 8. As illustrated in Figure 3-11, this sequence can be taken to infinity.

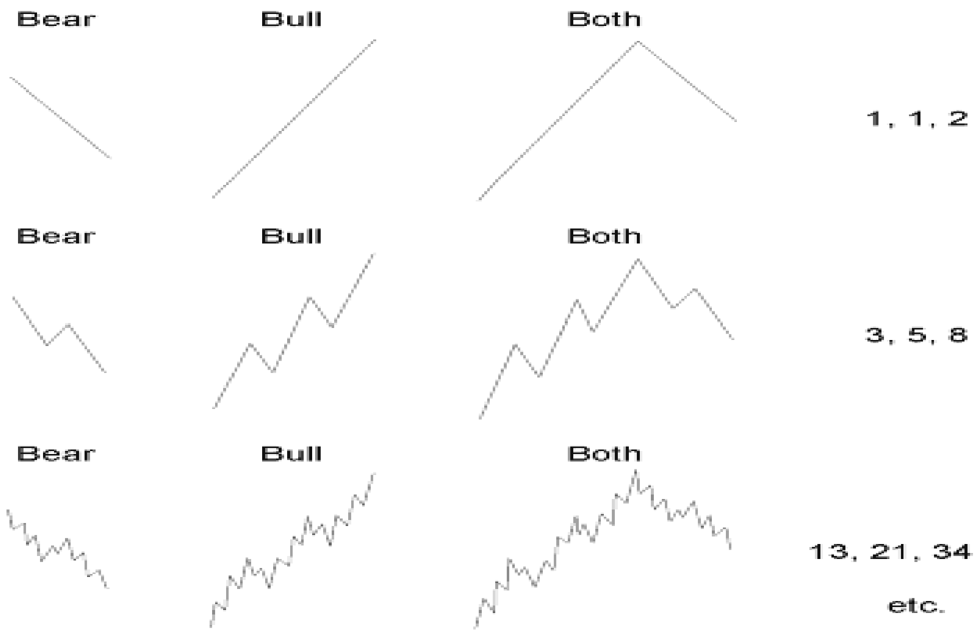
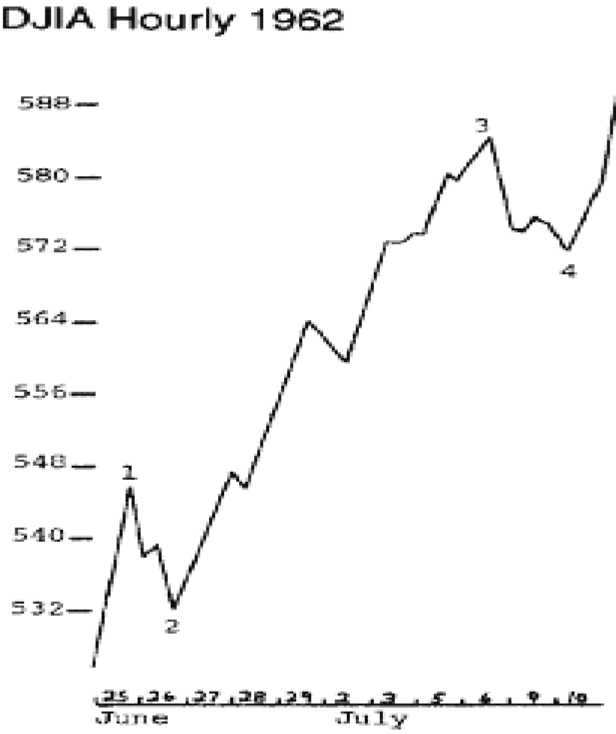


Figure 3-11

Lesson 19: Phi And The Stock Market

The stock market's patterns are repetitive (and fractal, to use today's terminology) in that the same basic pattern of movement that shows up in minor waves, using hourly plots, shows up in Supercycles and Grand Supercycles, using yearly plots. Figures 3-12 and 3-13 show two charts, one reflecting the hourly fluctuations in the Dow over a ten day period from June 25th to July 10th, 1962 and the other a yearly plot of the S&P 500 Index from 1932 to 1978 (courtesy of The Media General Financial Weekly). Both plots indicate similar patterns of movement despite a difference in the time span of over 1500 to 1. The long term formulation is still unfolding, as wave V from the 1974 low has not run its full course, but to date the pattern is along lines parallel to the hourly chart. Why? Because in the stock market, form is not a slave to the time element. Under Elliott's rules, both short and long term plots reflect a 5-3 relationship that can be aligned with the form that reflects the properties of the Fibonacci sequence of numbers. This truth suggests that collectively, man's emotions, in their expression, are keyed to this mathematical law of nature.



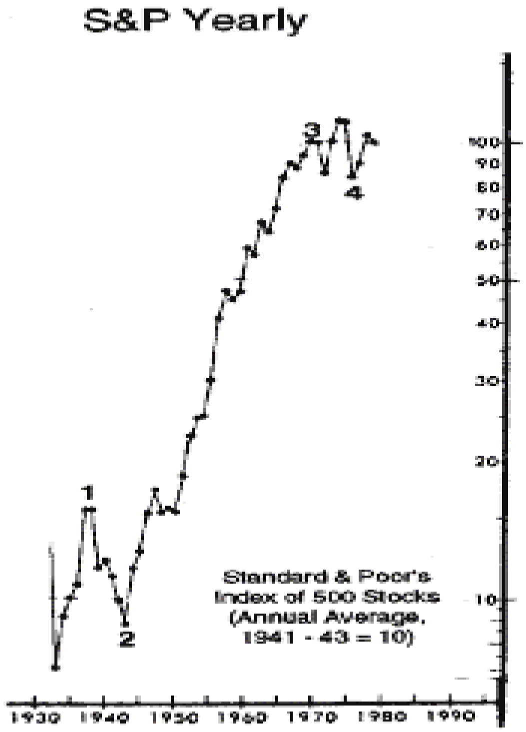


Figure 3-12 **Figure 3-13**

Now compare the formations shown in Figures 3-14 and 3-15. Each illustrates the natural law of the inwardly directed Golden Spiral and is governed by the Fibonacci ratio. Each wave relates to the previous wave by .618. In fact, the distances in terms of the Dow points themselves reflect Fibonacci mathematics. In Figure 3-14, showing the 1930-

1942 sequence, the market swings cover approximately 260, 160, 100, 60, and 38 points respectively, closely resembling the declining list of Fibonacci ratios: 2.618, 1.618, 1.00, .618 and .382.

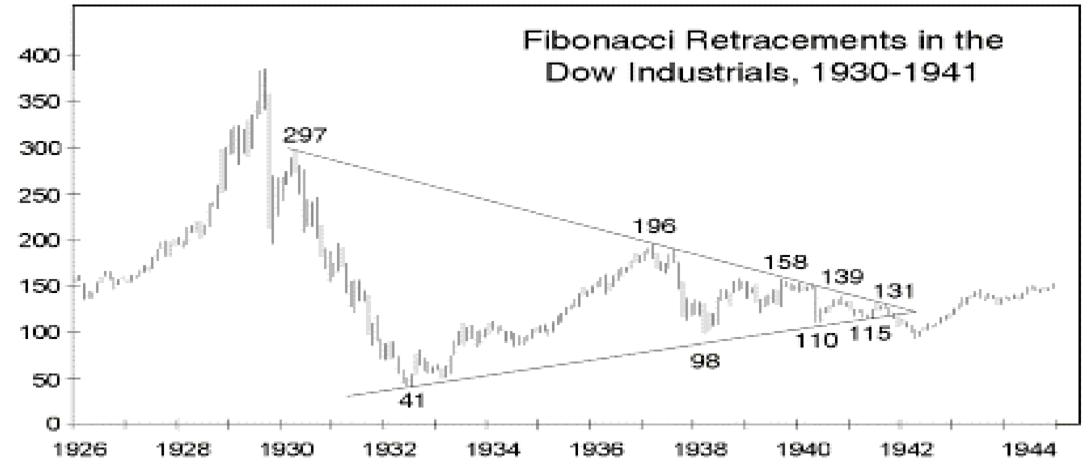


Figure 3-14

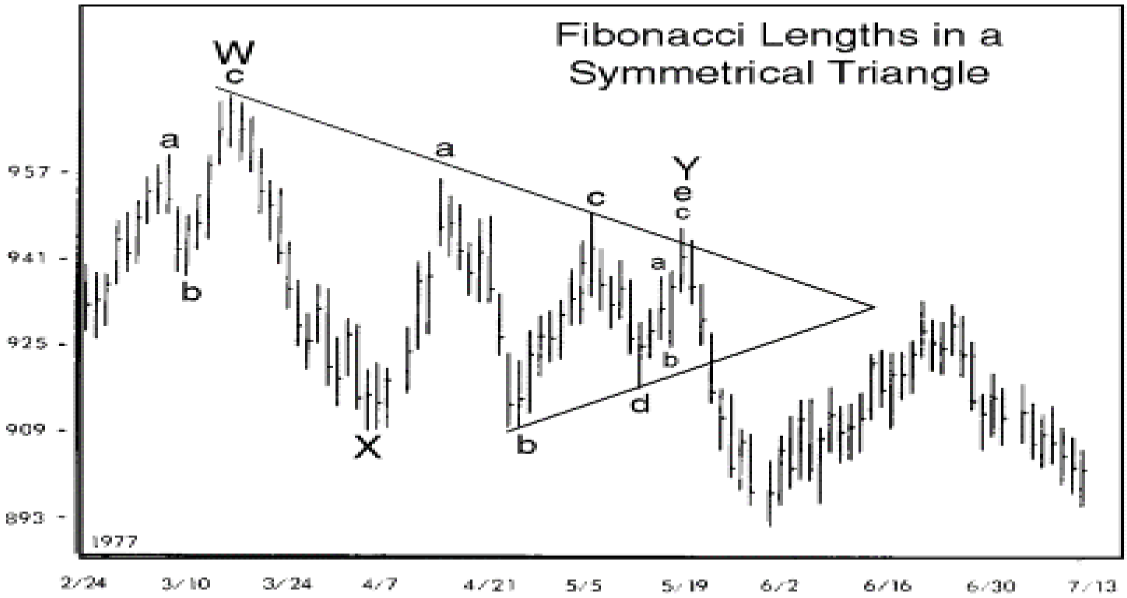


Figure 3-15

Starting with wave X in the 1977 upward correction shown in Figure 3-15, the swings are almost exactly 55 points (wave X), 34 points (waves A through C), 21 points (wave d), 13 points (wave a of e) and 8 points (wave b of e), the Fibonacci sequence itself. The total net gain from beginning to end is 13 points, and the apex of the triangle lies exactly on the level of the correction's beginning at 930, which is also the level of the peak of the subsequent reflex rally in June. Whether one takes the actual number of points in the waves as coincidence or part of the design, one can be certain that the precision manifest in the constant .618 ratio between each successive wave is not coincidence. Lessons 20 through 25 and 30 will elaborate substantially on the appearance of the Fibonacci ratio in market patterns.

Fibonacci Mathematics in the Structure of the Wave Principle

Even the ordered structural complexity of Elliott Wave forms reflects the Fibonacci sequence. There is 1 basic form: the five wave sequence. There are 2 modes of waves: motive (which subdivide into the cardinal class of waves, numbered) and corrective (which subdivide into the consonant class of waves, lettered). There are 3 orders of simple patterns of waves: fives, threes and triangles (which have characteristics of both fives and threes).

There are 5 families of simple patterns: impulse, diagonal triangle, zigzag, flat and triangle. There are 13 variations of simple patterns: impulse, ending diagonal, leading diagonal, zigzag, double zigzag, triple zigzag, regular flat, expanded flat, running flat, contracting triangle, descending triangle, ascending triangle and expanding triangle.

The corrective mode has two groups, simple and combined, bringing the total number of groups to 3. There are 2 orders of corrective combinations (double correction and triple correction), bringing the total number of orders to 5. Allowing only one triangle per combination and one zigzag per combination (as required), there are 8 families of corrective combinations in all: zig/flat, zig/tri., flat/flat, flat/tri., zig/flat/flat, zig/flat/tri., flat/flat/flat and flat/flat/tri., which brings the total number of families to 13. The total number of simple patterns and combination families is 21.

Figure 3-16 is a depiction of this developing tree of complexity. Listing permutations of those combinations, or further variations of lesser importance within waves, such as which wave, if any, is extended, which ways alternation is satisfied, whether an impulse does or does not contain a diagonal triangle, which types of triangles are in each of the combinations, etc., may serve to keep this progression going.

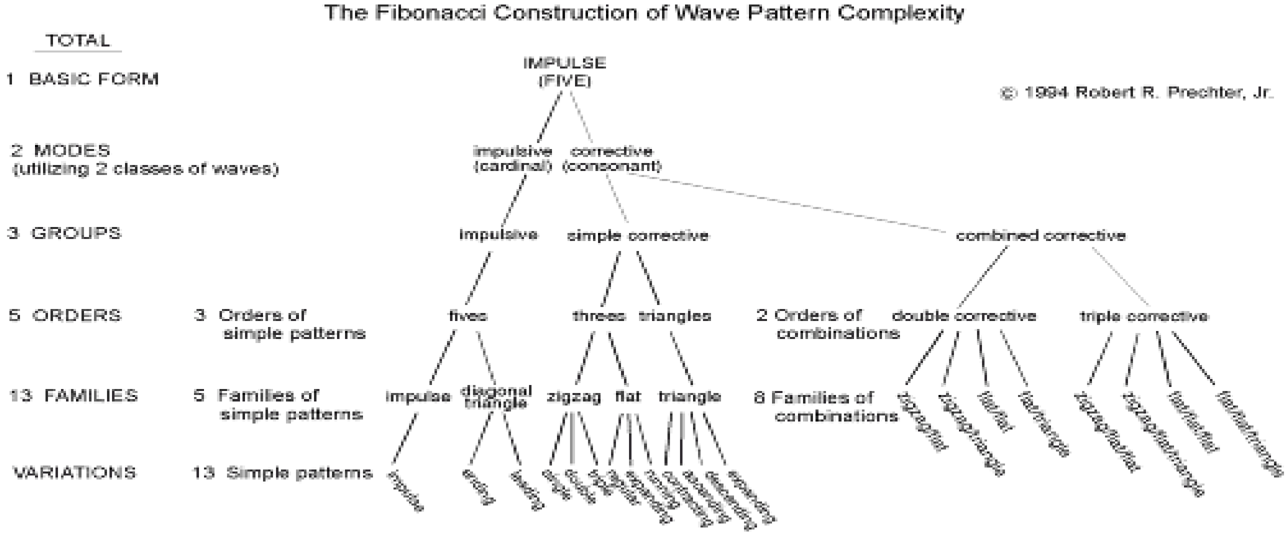


Figure 3-16

There may be an element of contrivance in this ordering process, as one can conceive of some possible variations in acceptable categorization.

Still, that a principle about Fibonacci appears to reflect Fibonacci is itself worth some reflection.

Phi and Additive Growth

As we will show in subsequent lessons, the spiral-like form of market action is repeatedly shown to be governed by the Golden Ratio, and even Fibonacci numbers appear in market statistics more often than mere chance would allow. However, it is crucial to understand that while the numbers themselves do have theoretic weight in the grand concept of the Wave Principle, it is the ratio that is the fundamental key to growth patterns of this type. Although it is rarely pointed out in the literature, the Fibonacci ratio results from this type of additive sequence no matter what two numbers start the sequence. The Fibonacci sequence is the basic additive sequence of its type since it begins with the number "1" (see Figure 3-17), which is the starting point of mathematical growth.

However, we may also take any two randomly selected numbers, such as 17 and 352, and add them to produce a third, continuing in that manner to produce additional numbers. As this sequence progresses, the ratio between adjacent terms in the sequence always approaches the limit phi very quickly. This relationship becomes obvious by the time the eighth term is produced (see Figure 3-18). Thus, while the specific numbers making up the Fibonacci sequence reflect the ideal progression of waves in markets, the Fibonacci ratio is a fundamental law of geometric progression in which two preceding units are summed to create the next. That is why this ratio governs so many relationships in data series relating to natural phenomena of growth and decay, expansion and contraction, and advancement and retreat.

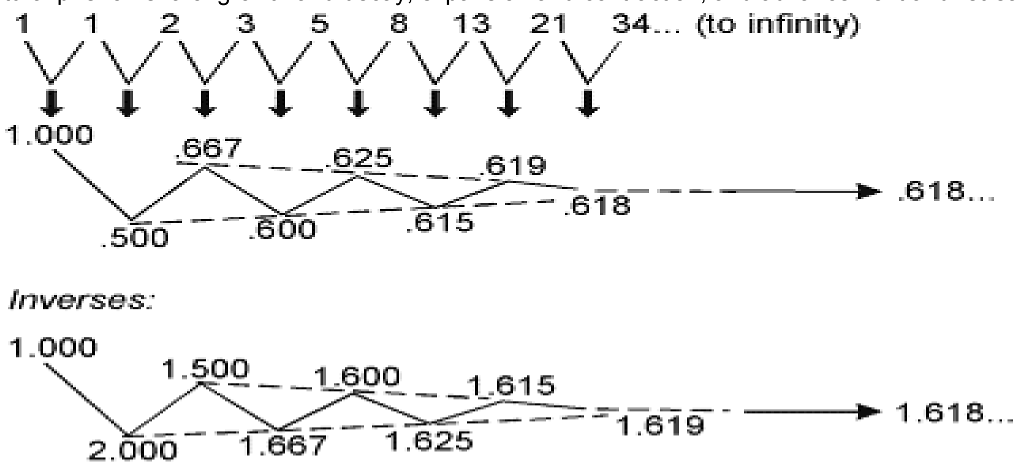


Figure 3-17

In its broadest sense, the Elliott Wave Principle proposes that the same law that shapes living creatures and galaxies is inherent in the spirit and activities of men en masse. The Elliott Wave Principle shows up clearly in the market because the stock market is the finest reflector of mass psychology in the world. It is a nearly perfect recording of man's social psychological states and trends, which produce the fluctuating valuation of his own productive enterprise, making manifest its very real patterns of progress and regress. What the Wave Principle says is that mankind's progress (of which the stock market is a popularly determined valuation) does not occur in a straight line, does not occur randomly, and does not occur cyclically. Rather, progress takes shape in a "three steps forward, two steps back" fashion, a form that nature prefers. In our opinion, the parallels between and Wave Principle and other natural phenomena are too great to be dismissed as just so much nonsense. On the balance of probabilities, we have come to the conclusion that there is a principle, everywhere present, giving shape to social affairs, and that Einstein knew what he was talking about when he said, "God does not play dice with the universe." The stock market is no exception, as mass behavior is undeniably linked to a law that can be studied and defined. The briefest way to express this principle is a simple mathematical statement: the 1.618 ratio.

The Desiderata, by poet Max Ehrmann, reads, "You are a child of the Universe, no less than the trees and the stars; you have a right to be here. And whether or not it is clear to you, no doubt the Universe is unfolding as it should." Order in life? Yes. Order in the stock market? Apparently.

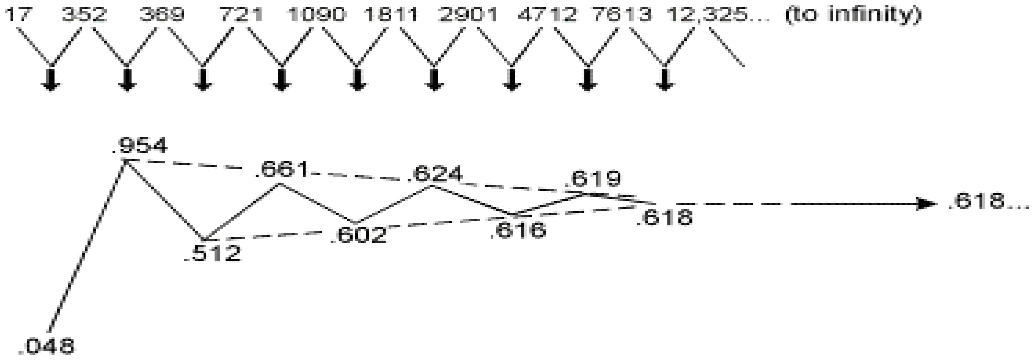


Figure 3-18

In 1939, Financial World magazine published twelve articles by R.N. Elliott entitled "The Wave Principle." The original publisher's note, in the introduction to the articles, stated the following:

During the past seven or eight years, publishers of financial magazines and organizations in the investment advisory field have been virtually flooded with "systems" for which their proponents have claimed great accuracy in forecasting stock market movements. Some of them appeared to work for a while. It was immediately obvious that others had no value whatever. All have been looked upon by The Financial World with great skepticism. But after investigation of Mr. R.N. Elliott's Wave Principle, The Financial World became convinced that a series of articles on this subject would be interesting and instructive to its readers. To the individual reader is left the determination of the value of the Wave Principle as a working tool in market forecasting, but it is believed that it should prove at least a useful check upon conclusions based on economic considerations. — The Editors of The Financial World

In the rest of this course, we reverse the editors' suggested procedure and argue that economic considerations at best may be thought of as an ancillary tool in checking market forecasts based entirely upon the Elliott Wave Principle.

Lesson 20: Introduction To Ratio Analysis

Ratio Analysis

Ratio analysis is the assessment of the proportionate relationship, in time and amplitude, of one wave to another. In discerning the working of the Golden Ratio in the five up and three down movement of the stock market cycle, one might anticipate that on completion of any bull phase, the ensuing correction would be three-fifths of the previous rise in both time and amplitude. Such simplicity is seldom seen. However, the underlying tendency of the market to conform to relationships suggested by the Golden Ratio is always present and helps generate the right look for each wave.

The study of wave amplitude relationships in the stock market can often lead to such startling discoveries that some Elliott Wave practitioners have become almost obsessive about its importance. Although Fibonacci time ratios are far less common, years of plotting the averages have convinced the authors that the amplitude (measured either arithmetically or in percentage terms) of virtually every wave is related to the amplitude of an adjacent, alternate and/or component wave by one of the ratios between Fibonacci numbers. However, we shall endeavor to present the evidence and let it stand or fall on its own merit.

The first evidence we found of the application of time and amplitude ratios in the stock market comes from, of all suitable sources, the works of the great Dow Theorist, Robert Rhea. In 1936, Rhea, in his book The Story of the Averages, compiled a consolidated summary of market data covering nine Dow Theory bull markets and nine bear markets spanning a thirty-six year time period from 1896 to 1932. He had this to say about why he felt it was necessary to present the data despite the fact that no use for it was immediately apparent:

Whether or not [this review of the averages] has contributed anything to the sum total of financial history, I feel certain that the statistical data presented will save other students many months of work.... Consequently, it seemed best to record all the statistical data we had collected rather than merely that portion which appeared to be useful.... The figures presented under this heading probably have little value as a factor in estimating the probable extent of future movements; nevertheless, as a part of a general study of the averages, the treatment is worthy of consideration.

One of the observations was this one:

The footings of the tabulation shown above (considering only the industrial average) show that the nine bull and bear markets covered in this review extended over 13,115 calendar days. Bull markets were in progress 8,143 days, while the remaining 4,972 days were in bear markets. The relationship between these figures tends to show that bear markets run 61.1 percent of the time required for bull periods.

And finally, Column 1 shows the sum of all primary movements in each bull (or bear) market. It is obvious that such a figure is considerably greater than the net difference between the highest and lowest figures of any bull market. For example, the bull market discussed in Chapter II started (for Industrials) at 29.64 and ended at 76.04, and the